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Seokkwan Yoon

Sverdrup Technology, Inc.
Lewis Research Center
Cleveland, Ohio

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Seokkwan Yoon
Sverdrup Technology, Inc.
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

A new Navier-Stokes solver is developed by combining the efficiency of the LU-SSOR scheme and the accuracy of the flux-limited dissipation scheme. Application to laminar and turbulent flows and hypersonic flows proves the reliability of the new algorithm.

1. INTRODUCTION

An obvious way to accelerate convergence to a steady state is to increase the size of time step. Implicit schemes can take larger time steps than explicit schemes when the time step of an explicit scheme is restricted by stability rather than accuracy. Since the unfactored implicit scheme produces a large block banded matrix which is very costly to invert, the factored implicit schemes have been popular. However, using a large time step resulted in a large factorization error. Moreover, inherent stability problems occurred when the number of factors were greater than or equal to three. Nevertheless, several factored implicit schemes have proved to be useful [1 to 5].

An alternative way to get a steady-state solution is to solve the steady governing equations by the Newton method. Because of the rapid growth of the operation count with the number of mesh cells, the system was solved indirectly by the relaxation methods. Recently, a new relaxation method, the LU-SSOR scheme, was developed by combining the advantages of LU factorization and Gauss-Seidel relaxation. The vectorizable LU-SSOR scheme, which is based on central differences, requires scalar diagonal inversions [6].

It has long been recognized that upwind differencing can eliminate undesirable oscillations near shock waves. Stemming from the mathematical theory of scalar conservation laws, Harten proposed the concept of total variation diminishing (TVD) schemes [7]. TVD schemes preserve the monotonicity of an initially monotone profile, because the total variation would increase if the profile ceased to be monotone. Second order schemes can be constructed by using multipoint extrapolation formulas to estimate the numerical flux, or by adding higher order dissipative terms. In either case flux limiters are then needed to control the signs of the coefficients of a semi-discrete approximation to the hyperbolic system. However, TVD schemes in two or more space dimensions are at most first order accurate. Nevertheless, two-dimensional results using one-dimensional second order TVD schemes and dimensional splitting showed sharp resolution of discontinuities without oscillations. Jameson constructed an efficient scheme which can be converted to a TVD scheme in the case of a scalar conservation law using flux limited dissipation [8].

In this paper, a new Navier-Stokes solver is developed by combing the efficiency of the LU-SSOR scheme and the accuracy of the flux limited dissipation scheme. Numerical examples include laminar and turbulent airfoils and a hypersonic inlet.

2. THE NAVIER-STOKES EQUATIONS

The Navier-Stokes equations represent gas flow in thermodynamic equilibrium. Let t, ρ, E, T, and p be time, density, total energy, temperature, and pressure; u and v Cartesian velocity components; and x and y Cartesian coordinates. Then for a two-dimensional flow these equations can be written as

$$\frac{\partial W}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = \frac{\partial F_v}{\partial x} + \frac{\partial G_v}{\partial y} \tag{1}$$

where W is the vector of dependent variables, and F and G are convective flux vectors:

$$\left.\begin{aligned} W &= (\rho,\ \rho u,\ \rho v,\ \rho E)^* \\ F &= [\rho u,\ \rho u^2 + p,\ \rho v u,\ u(\rho E + p)]^* \\ G &= [\rho v,\ \rho u v,\ \rho v^2 + p,\ v(\rho E + p)]^* \end{aligned}\right\} \tag{2}$$

Here $*$ denotes the transpose of a matrix. The flux vectors for the viscous terms are

$$F_v = \left(0,\ \tau_{xx},\ \tau_{xy},\ u\tau_{xx} + v\tau_{xy} + k\,\frac{\partial T}{\partial x}\right)^*$$

$$G_v = \left(0,\ \tau_{xy},\ \tau_{yy},\ u\tau_{xy} + v\tau_{yy} + k\,\frac{\partial T}{\partial y}\right)^*$$

Here the viscous stresses are

$$\tau_{xx} = 2\mu u_x - \frac{2}{3}\,\mu(u_x + v_y)$$

$$\tau_{xy} = \mu(u_y + v_x)$$

and

$$\tau_{yy} = 2\mu v_y - \frac{2}{3}\mu(u_x + v_y)$$

where μ is the coefficient of viscosity and k is the coefficient of thermal conductivity.

The pressure is obtained from the equation of state:

$$p = \rho(\gamma - 1)\left\{E - \frac{1}{2}(u^2 + v^2)\right\} \tag{3}$$

where γ is the ratio of specific heats.

3. THE LU-SSOR SCHEME

A prototype implicit scheme for a system of nonlinear hyperbolic equations such as the Euler equations can be formulated as

$$W^{n+1} = W^n - \beta\Delta t\left[D_x F(W^{n+1}) + D_y G(W^{n+1})\right] - (1-\beta)\Delta t\left[D_x F(W^n) + D_y G(W^n)\right] \tag{4}$$

where D_x and D_y are central difference operators that approximate $\partial/\partial x$ and $\partial/\partial y$. Here n denotes the time level. An enormous number of computations must be performed when the scheme is in this form because coupled nonlinear equations must be solved at each time step. Let the Jacobian matrices be

$$\left.\begin{aligned} A &= \frac{\partial F}{\partial W} \\ B &= \frac{\partial G}{\partial W} \end{aligned}\right\} \tag{5}$$

and let the correction be

$$\Delta W = W^{n+1} - W^n$$

The scheme can be linearized by setting

$$F(W^{n+1}) = F(W^n) + A\delta W + O(\|\delta W\|^2)$$

$$G(W^{n+1}) = G(W^n) + B\delta W + O(\|\delta W\|^2)$$

where O is the order of the enclosed terms, and dropping terms of the second and higher orders to yield

$$[I + \beta\Delta t(D_x A + D_y B)]\delta W + \Delta t\ R = 0 \tag{6}$$

where I is the identity matrix and R is the residual

$$R = D_x F(W^n) + D_y G(W^n)$$

If a constant $\beta = 1/2$, the scheme remains second-order accurate in time; for other values of β, the time accuracy drops to first order.

The unfactored implicit scheme (Eq. (6)) produces a large block banded matrix that can be inverted only by performing a great many computations. In addition, a large amount of storage is required. If $\beta = 1$ the scheme reduces to a Newton iteration in the limit $\Delta t \to \infty$:

$$(D_x A + D_y B)\delta W + R = 0 \tag{7}$$

A diagonally dominant form of Eq. (7) is

$$(D_x^- A^+ + D_x^+ A^- + D_y^- B^+ + D_y^+ B^-)\delta W + R = 0 \tag{8}$$

where D_x^- and D_y^- are backward difference operators and D_x^+ and D_y^+ are forward difference operators. Here, two-point operators are used for steady flow calculations. A^+, A^-, B^+, and B^- are constructed so that the eigenvalues of "+" matrices are nonnegative and those of "-" matrices are nonpositive:

$$\left.\begin{aligned} A^+ &= \frac{1}{2}(A + r_A I) \\ A^- &= \frac{1}{2}(A - r_A I) \\ B^+ &= \frac{1}{2}(B + r_B I) \\ B^- &= \frac{1}{2}(B - r_B I) \end{aligned}\right\} \tag{9}$$

where

$$\left.\begin{aligned} r_A &\geq \max\ (|\lambda_A|) \\ r_B &\geq \max\ (|\lambda_B|) \end{aligned}\right\} \tag{10}$$

Here, λ_A and λ_B represent eigenvalues of Jacobian matrices.

Then, the LU-SSOR scheme for approximate Newton iteration becomes

$$(D_x^- A^+ + D_y^- B^+ - A^- - B^-)(D_x^+ A^- + D_y^+ B^- + A^+ + B^+)\delta W = -(r_A + r_B)(D_x F + D_y G) \qquad (11)$$

which can be inverted in two steps as

$$(D_x^- A^+ + D_y^- B^+ - A^- - B^-)\,\delta W^* = -(r_A + r_B)(D_x F + D_y G)$$

$$(D_x^+ A^- + D_y^+ B^- + A^+ + B^+)\,\delta W = \delta W^* \qquad (12)$$

For the Navier-Stokes equations, F and G are replaced by $F - F_v$ and $G - G_v$ in Eq. (11). That is,

$$\left(D_x^- A^+ + D_y^- B^+ - A^- - B^-\right)\left(D_x^+ A^- + D_y^+ B^- + A^+ + B^+\right)\delta W$$
$$= -(r_A + r_B)\left\{D_x(F - F_v) + D_y(G - G_v) + \text{flux limited dissipation terms}\right\} \qquad (13)$$

Since one-sided difference schemes are naturally dissipative, no implicit smoothing is required on the left side. Only adaptive dissipation terms are explicitly added to the residual on the right side. It is interesting to note that the present numerical method eliminates the need for block diagonal inversions without using the diagonalization process. This is an especially desirable feature for the analysis of hypersonic reacting flows. The LU family of algorithms are fully vectorizable along i + j = constant lines on a vector computer.

4. JAMESON'S TVD SCHEME

Consider the scalar conservation law

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} f(u) = 0 \qquad (14)$$

It is well known that the total variation

$$TV = \int_{-\infty}^{\infty} \left|\frac{\partial u}{\partial x}\right| dx \qquad (15)$$

cannot increase. Suppose now that a multipoint semi-discrete approximation to equation (14) is expressed in the form

$$\frac{du_i}{dt} = \sum_{q=-Q}^{Q-1} c_q(i)(u_{i-q} - u_{i-q-1}) \tag{16}$$

The discrete total variation is

$$TV = \sum_{i=-\infty}^{\infty} |u_i - u_{i-1}|$$

It can be shown that this will not increase if and only if

$$c_{-1}(i-1) \geq c_{-2}(i-2) \ldots \geq c_{-Q}(i-Q) \geq 0$$

and (17)

$$-c_0(i) \geq -c_1(i+1) \ldots \geq -c_{Q-1}(i+Q-1) \geq 0$$

Now consider the scheme

$$\frac{du_i}{dt} + \frac{1}{\Delta x}(h_{i+1/2} - h_{i-1/2}) = 0 \tag{18}$$

where $h_{i+1/2}$ is an approximation to the flux across the boundary between the $(i+1)^{st}$ and i^{th} cells. Denoting $f(u_i)$ by f_i, define the numerical flux as

$$h_{i+1/2} = \frac{1}{2}(f_{i+1} + f_i) + d_{i+1/2} \tag{19}$$

where $d_{i+1/2}$ is a dissipative flux.

$$d_{i+1/2} = \alpha_{i+1/2}\left\{B(e_{i+3/2}, e_{i+1/2}) - 2e_{i+1/2} + B(e_{i+1/2}, e_{i-1/2})\right\} \tag{20}$$

where $e_{i+1/2} = \rho_{i+1} - \rho_i$ for example.

B (p,q) is known as Roe's min mod function which can be defined as

$$B(p,q) = \left\{s(p) + s(q)\right\} \min(|P|,|q|) \tag{21}$$

where

$$s(p) = \begin{cases} \frac{1}{2} & \text{if } P \geq 0 \\ -\frac{1}{2} & \text{if } P < 0 \end{cases}$$

Here

$$\alpha_{i+1/2} = \min\,(1/2,\ k_o+k_1\bar{\nu}_{i+1/2})(R_{i+1} - R_i) \tag{22}$$

where k_o and k_1 are the constants and

$$\bar{\nu}_{1+1/2} = \max(\nu_{i+2}, \nu_{i+1}, \nu_i, \nu_{i-1})$$

$$\nu_i = \left| \frac{P_{i+1} - 2P_i + P_{i-1}}{P_{i+1} + 2P_i + P_{i-1}} \right|$$

The spectral radius R can be estimated as the value of

$$R = |\ \Delta y u - \Delta x v\ | + c\sqrt{\Delta x^2 + \Delta y^2}$$

on the edge separating cells (i+1,j) and (i,j) in two-dimensions where c is the speed of sound.

5. RESULTS

The first test case was for viscous laminar flow past the NACA 0012 airfoil at Mach 0.5, Reynolds number 5000, and zero angle of attack. The adiabatic wall boundary condition was used at the body surface. Calculations were performed on a stretched 192 by 48 C-mesh. Figure 1 shows the Mach number contours while Fig. 2 shows velocity vectors.

The next case was for viscous turbulent flow past the RAE 2822 airfoil at Mach 0.73, Reynolds number 6.5 million, and 2.79° angle of attack. The Reynolds - averaged Navier-Stokes equations were solved using a Baldwin-Lomax turbulence model. Transition was fixed at 3 percent chord. Mach number contours are shown in Fig. 3 where the dashed line denotes the sonic line. It is interesting to note that present TVD scheme produces identical solutions to the adaptive dissipation results in Ref. 6 in the case of the weak shock waves.

The last cases were for inviscid hypersonic flows past an inlet. Calculations were done on a uniform 54 by 32 H-mesh. Figures 4 and 5 show the Mach number contours for Mach 5 and 10 flows respectively. When compared to the results of Ref. 9 using the adaptive dissipation, present TVD scheme results show sharper resolution and significant improvement in both local and global accuracy.

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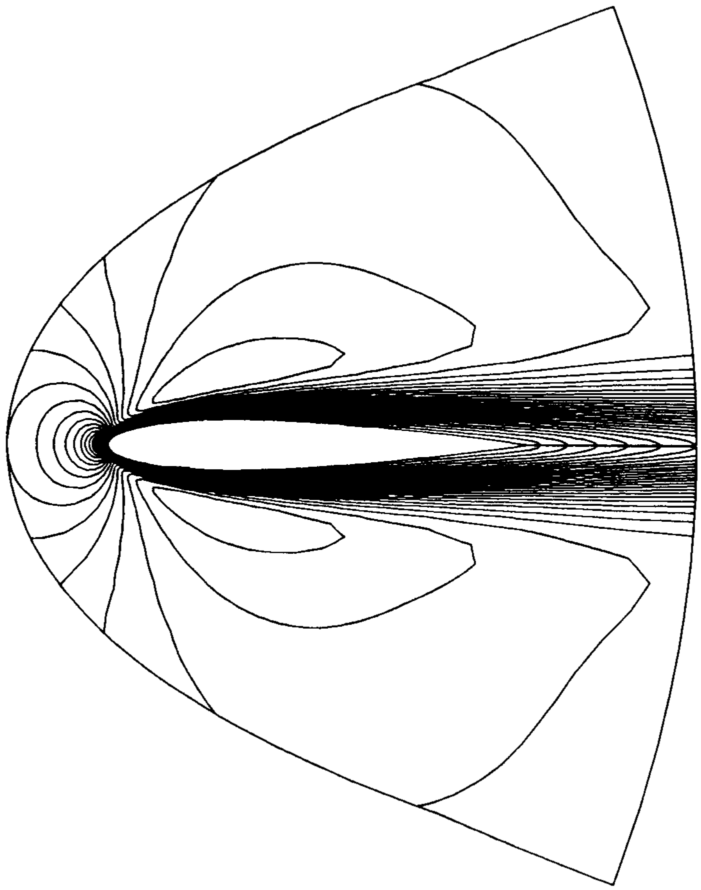

FIGURE 1. - MACH NUMBER CONTOURS FOR VISCOUS LAMINAR FLOW.

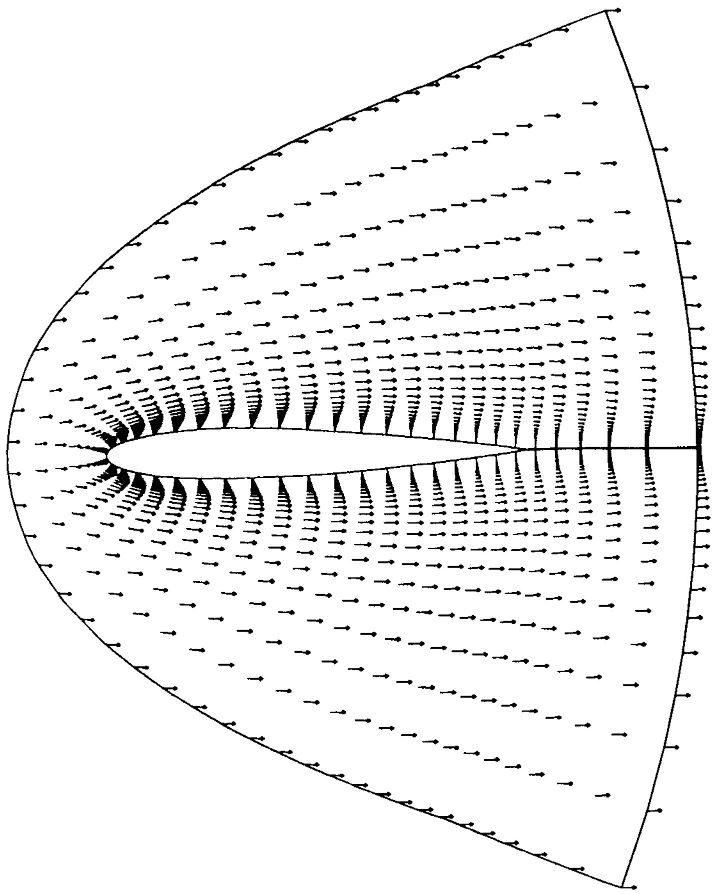

FIGURE 2. - VELOCITY VECTORS FOR VISCOUS LAMINAR FLOW.

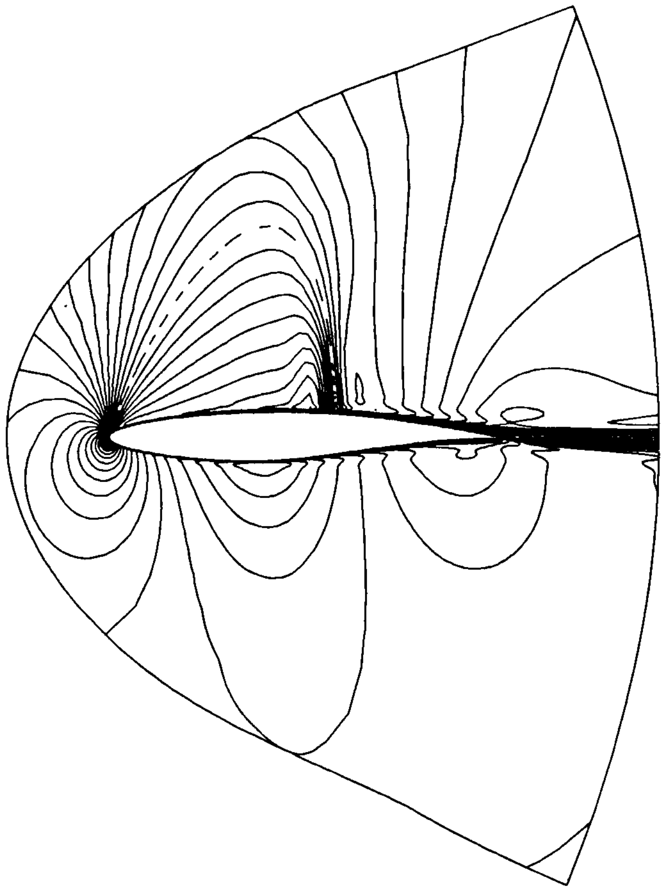

FIGURE 3. - MACH NUMBER CONTOURS FOR VISCOUS TURBULENT FLOW.

Figure 4. - Mach number contours for Mach 5 inlet.

Figure 5. - Mach number contours for Mach 10 inlet.

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16. Abstract

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